Improvement of Convergence Characteristics of 1-D Dynamic Magnetic Field Analysis with Hysteresis for Iron Loss Estimation

Satoshi Doi[1,2], Tetsuya Aoki[2], Keiichi Okazaki[2], Yasuhito Takahashi[1], and Koji Fujiwara[1]

[1]Department of Electrical Engineering, Doshisha University, Kyoto 610-0321, Japan, euo1301@mail4.doshisha.ac.jp
[2]Material Engineering Department, DENSO Co., Kariya 448-8661, Japan, satoshi_doi@denso.co.jp

In previous study on precise computation methods of iron loss, it has been shown that 1-dimensional (1-D) dynamic magnetic field analysis with a hysteresis in post-processing is effective. However, there is a problem that the hysteresis makes the convergence of nonlinear iteration unstable. So this paper proposes a stabilization method for 1-D dynamic magnetic field analysis with a hysteresis. The proposed method is characterized as an approach to improving the initial value and the step size in Newton-Raphson method (NR). As a result of the study, it is shown that this method can improve the convergence characteristic of nonlinear iteration. Therefore it can be expected that stable convergence for harmonic magnetic flux can be maintained in electric motors.

***Index Terms*— Eddy current, finite element method, initial value, iron loss, magnetic hysteresis, Newton-Raphson method.**

I. Introduction

The demand for compact and high efficient automotive electric motors has been increasing, so it has become necessary to establish the limitations in design of electric motors. Therefore, it has become important to accurately compute iron loss.

Magnetic flux density of each part in electric motor is usually computed by 2-dimensional (2-D) static magnetic field analysis using initial magnetization curves. In [1], iron loss is computed directly by 1-D dynamic magnetic field analysis with hysteresis as a post-processing of the 2-D magnetic field analysis. This method has possibility to compute the harmonic loss which has been estimated with low accuracy by using Steinmetz empirical formula, although this method requires more computation time than the Steinmetz method. In addition, there is a problem that the convergence of nonlinear iteration becomes unstable because of complicated hysteretic properties. So the purpose of this paper is to improve the convergence of 1-D dynamic magnetic analysis with the hysteresis by improving the step size in NR method and the initial value.

II. Computation Method of Magnetic Loss

A. Eddy current loss calculation

Eddy current is computed by 1-D dynamic magnetic field analysis as a post-processing of usual magnetic field analyses [2]. It's discretized by Galerkin method as follows:

$$\begin{cases} G_{ix} = \int \left(H_y \dfrac{\partial N_i}{\partial z} + \sigma N_i \dfrac{\partial A_x}{\partial t} \right) dz = 0 \\ G_{iy} = \int \left(-H_x \dfrac{\partial N_i}{\partial z} + \sigma N_i \dfrac{\partial A_y}{\partial t} \right) dz = 0 \end{cases} \tag{1}$$

where σ is electric conductivity, N is interpolation function. Boundary conditions are shown as follows:

$$\begin{cases} A_x(t,0) = 0, \quad A_x(t,h/2) = B_y^{2\mathrm{D}}(t)h/2 \\ A_y(t,0) = 0, \quad A_y(t,h/2) = -B_x^{2\mathrm{D}}(t)h/2 \end{cases} \tag{2}$$

where h is the thickness of a steel sheet, $B^{2\mathrm{D}}$ is magnetic flux density obtained by 2-D magnetic field analysis.

B. Magnetic hysteresis loss calculation

In this paper, magnetic hysteresis is represented by isotropic vector play model [3] as follows:

$$\boldsymbol{H} = \sum_{n=1}^{N_\mathrm{p}} f_{\zeta n}\left(\left| \boldsymbol{P}_{\zeta n} \right| \right) \frac{\boldsymbol{P}_{\zeta n}}{\left| \boldsymbol{P}_{\zeta n} \right|} \tag{3}$$

$$\boldsymbol{P}_{\zeta n} = \boldsymbol{B} - \zeta_n \frac{\boldsymbol{B} - \boldsymbol{P}_{0,\zeta n}}{\max\left(\left| \boldsymbol{B} - \boldsymbol{P}_{0,\zeta n} \right|, \zeta_n \right)} \tag{4}$$

$$\zeta_n = B_\mathrm{s}(n-1)/N_\mathrm{P} \quad n = 1, \cdots, N_\mathrm{P} \tag{5}$$

where B_s is the maximum measurable magnetic flux density, N_p is the number of hysterons, $f_{\zeta n}$ is the shape function for the play hysteron operator $|\boldsymbol{P}_{\zeta n}|$. The subscript 0 denotes the value at the previous time step.

III. Stabilization Method

A. Step size improvement

Reference [4] proposed to improve the convergence of nonlinear iteration by determining the step size α based on minimizing energy functional using line search (functional NR). In addition, [5] proposed to use the NR method for α as follows (exact NR):

$$\delta \boldsymbol{A}^{(k)} \frac{\partial \boldsymbol{G}^{(k+1)\mathrm{T}}}{\partial \alpha^{(k)}} \delta \alpha^{(k)} = -\boldsymbol{G}^{(k+1)\mathrm{T}} \delta \boldsymbol{A}^{(k)}, \quad \alpha^{(k)} = \alpha^{(k)} + \delta \alpha^{(k)} \tag{6}$$

where k is the number of nonlinear iterations, $\partial \boldsymbol{G} / \partial \alpha$ in 1-D dynamic magnetic field analysis is represented as follows:

$$\frac{\partial G_{ix}^{(k+1)}}{\partial \alpha^{(k)}} = \int \left\{ \frac{\partial N_i}{\partial z} \left(\frac{\partial H_y^{(k+1)}}{\partial B_y^{(k+1)}} \delta B_y^{(k)} + \frac{\partial H_y^{(k+1)}}{\partial B_x^{(k+1)}} \delta B_x^{(k)} \right) + \frac{\sigma N_i}{\Delta t} \delta A_x^{(k)} \right\} dz \tag{7}$$

$$\frac{\partial G_{iy}^{(k+1)}}{\partial \alpha^{(k)}} = \int \left\{ -\frac{\partial N_i}{\partial z} \left(\frac{\partial H_x^{(k+1)}}{\partial B_y^{(k+1)}} \delta B_y^{(k)} + \frac{\partial H_x^{(k+1)}}{\partial B_x^{(k+1)}} \delta B_x^{(k)} \right) + \frac{\sigma N_i}{\Delta t} \delta A_y^{(k)} \right\} dz \tag{8}$$

$$\delta B_y^{(k)} \equiv \frac{\partial \delta A_x^{(k)}}{\partial z}, \quad \delta B_x^{(k)} \equiv -\frac{\partial \delta A_y^{(k)}}{\partial z}, \quad |\delta B| = \sqrt{\delta B_x^{(k)2} + \delta B_y^{(k)2}} \tag{9}$$

The convergence criterion is $|\delta B| < 1\mathrm{mT}$ in this paper.

B. Initial value improvement

Generally, the closer the initial value $\boldsymbol{A}^{(0)}$ in the NR method is to the solution, the more the number of iterations decreases.

Therefore, we examine the method of determining the initial value in below.

Method (i): **0** which is the basic way.

Method (ii): solutions in the previous time step (A_0).

Method (iii): $A_x^{(0)} = B_y^{2D} z$, $A_y^{(0)} = -B_y^{2D} z$, which correspond to the solutions of (1) in static magnetic field analysis

Method (iv): analytical solutions for the linear equation (10). The time-derivative term is discretized by the backward difference method as follows:

$$\nu \frac{\partial^2 A}{\partial z^2} = \sigma \frac{\partial A}{\partial t}, \quad \Rightarrow \frac{\partial^2 A}{\partial z^2} - FA = -FA_0, \quad F = \frac{\sigma}{\nu \Delta t} \tag{10}$$

where A_0 is approximated by a polynomial based on the least-square method to analytically solve the differential equation. In this paper, the degree m of the polynomial is the smallest value which satisfies that the determination coefficient R is over 0.99. The particular solution with the boundary condition (2) is represented as follows :

$$A_x^{(0)} = \frac{e^{\sqrt{F}z} - e^{-\sqrt{F}z}}{e^{\sqrt{F}h/2} - e^{-\sqrt{F}h/2}} \left\{ -\frac{B_y^{2D} h}{2} - \sum_{i=1}^{m} Q_{x,2i-1} \left(\frac{h}{2}\right)^{2i-1} \right\} + \sum_{i=1}^{m} Q_{x,2i-1} z^{2i-1} \tag{11}$$

$$A_y^{(0)} = \frac{e^{\sqrt{F}z} - e^{-\sqrt{F}z}}{e^{\sqrt{F}h/2} - e^{-\sqrt{F}h/2}} \left\{ \frac{B_x^{2D} h}{2} - \sum_{i=1}^{m} Q_{y,2i-1} \left(\frac{h}{2}\right)^{2i-1} \right\} + \sum_{i=1}^{m} Q_{y,2i-1} z^{2i-1} \tag{12}$$

where Δt is the time interval, Q_{2i-1} is represented as

$$\begin{cases} Q_{2i-1} = C_{2i-1} & (i = m) \\ Q_{2i-1} = C_{2i-1} + \dfrac{2i(2i+1)}{F} Q_{2i+1} & (i < m) \end{cases} \tag{13}$$

IV. Analysis Conditions

In order to evaluate the effect of the stabilization method, the boundary condition B^{2D} is given a sine-wave.

TABLE I

Analysis Conditions

Amp. of magnetic flux density[T]	B_x^{2D}	1.0
	B_y^{2D}	1.0
Phase difference between B_x^{2D} and B_y^{2D}[deg.]		45
Frequency[Hz]		1, 100, 10k
Magnetic property		equivalent to JIS 50A290
Electrical property[μΩ・cm]		47
Thickness h [mm]		0.5
Number of cycles		2
Number of time steps		256
Number of elements (z-axis)		20
Linear solver		ILUBiCGStab ($\varepsilon_{cg} = 10^{-6}$)

V. Results

Table II shows the average number of iterations and CPU time required to obtain the convergence. The Normal NR with methods (i) and (ii) do not converge, but the functional and exact NR in each condition do. Particularly, the exact NR with method (iv) requires the least iteration in each frequency.

TABLE II

Number of Nonlinear Iterations and CPU Time

(a) Frequency : 1Hz

Initial value	Normal NR (α=1)		Functional NR		Exact NR	
	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]
(i)	no conv.	-	7.36	0.83	3.00	0.54
(ii)	no conv.	-	7.64	0.81	6.44	0.89
(iii)	2.00	0.15	2.00	0.31	2.00	0.32
(iv)	2.00	0.15	2.00	0.28	2.00	0.29

(b) Frequency : 100Hz

Initial value	Normal NR (α=1)		Functional NR		Exact NR	
	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]
(i)	no conv.	-	7.91	0.98	5.18	0.95
(ii)	no conv.	-	8.13	1.05	7.21	1.23
(iii)	3.63	0.22	3.56	0.53	3.53	0.47
(iv)	3.02	0.20	3.03	0.45	2.97	0.42

(c) Frequency : 10kHz

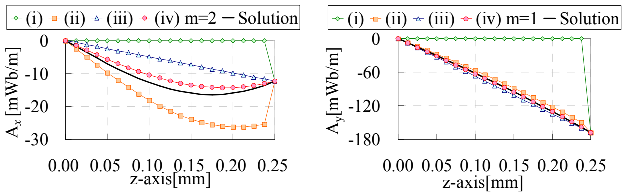
Initial value	Normal NR (α=1)		Functional NR		Exact NR	
	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]	Avg. ite.	Total time[s]
(i)	no conv.	-	12.9	1.54	12.2	2.05
(ii)	no conv.	-	7.73	1.00	7.44	1.26
(iii)	6.77	0.48	7.05	0.92	5.99	0.92
(iv)	5.98	0.40	5.36	0.79	5.19	0.80

computer used : Core i5-2400 / 3.1GHz with 6 GB RAM

Fig.1 shows initial value distributions. In method (iv), the initial value $\boldsymbol{A}^{(0)}$ is the closest to the solution.

Fig.2 shows the convergence characteristic. The residual norm $\|\boldsymbol{G}\|_2$ of method (iv) is the smallest in the first iteration, and decreases gradually up to convergence.

In the full paper, the stabilization method for nonlinear iteration will be considered in detail, and the precision of the proposed iron loss calculation method will be verified in the analysis of an actual motor.



(a) Vector potential $A_x^{(0)}$ (b) Vector potential $A_y^{(0)}$

Fig. 1. Comparison of initial value $\boldsymbol{A}^{(0)}$ (100Hz, time step = 128).

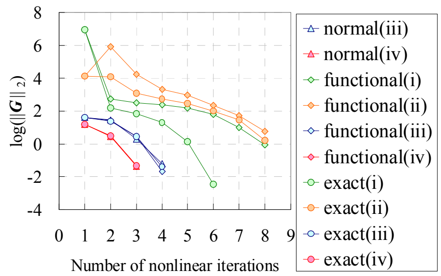


Fig. 2. Convergence characteristic (100Hz, time step = 128).

VI. References

[1] Y. Takeda, Y. Takahashi, K. Fujiwara, A. Ahagon, and T. Matsuo, “Iron Loss Estimation Method for Rotating Machines Taking Account of Hysteretic Property,” *IEEE Trans. Magn.*, to be published.

[2] K. Yamazaki and N. Fukushima, “Iron Loss Model for Rotating Machines Using Direct Eddy Current Analysis in Electrical Steel Sheets,” *IEEE Trans. Energy Conversion.*, vol. 25, no. 3, pp. 633-641, 2010.

[3] T. Matsuo and M. Shimasaki, “Two Types of Isotropic Vector Play Models and Their Rotational Hysteresis Losses,” *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 898-901, 2008.

[4] K. Fujiwara, Y. Okamoto, A. Kameari, and A. Ahagon, “The Newton-Raphson Method Accelerated by Using a Line Search – Comparison Between Energy Functional and Residual Minimization,” *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1724-1727, 2005.

[5] Y. Okamoto, K. Fujiwara, and R. Himeno, “Exact Minimization of Energy Functional Newton-Raphson Method with Line-Search Technique,” *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1288-1299, 2008.